**STATISTICS ASSIGNMENT-4**

**Q1. Is it possible that an event is independent of itself? If so, when?**

**Answer:**

The only events that are independent of themselves are those with probability either 0 or 1.

That follows from the fact that a number is its own square if and only if it's either 0 or 1. The only way a random variable X can be independent of itself is if for every measurable set A, either Pr (X∈A)=1 or Pr (X∈A)=0.

**Q2. Is it always true that if A and B are independent events, then Ac and Bc are independent events? Show that it is, or give a counterexample.**

**Answer:**

Events A and B are independent if: knowing whether A occured does not change the probability of B.

Mathematically, can say in two equivalent ways:

P(B|A) = P(B) P(A and B)

= P(B ∩ A) = P(B) × P(A).

Important to distinguish independence from mutually exclusive which would say B ∩ A is empty (cannot happen).

**Example**: Deal 2 cards from deck.

A - first card is Ace.

C - second card is Ace.

P(C|A) = 3/51

P(C) = 4/52.

So A and C are dependent.

**Example**: Throw 2 dice

A - First die lands 1.

B - Second die shows larger number than first die.

C - Both dice show same number P(B|A) = 5/6 P(B) =? = 15/36 by counting.

So A and B dependent.

P(C|A) = 1/6 P(C) = 6/36 = 1/6

So A and C independent.

Note 1: here B and C are mutually exclusive.

Note 2: writing B’ = ”second die shows smaller number than first die ”

P(B’ ) = P(B) by symmetry P(B ∪ B’ ) = P(Cc )=1 − P(C) = 5/6

giving a “non-counting” argument that P(B) = 5/12 .

**Example:** Deal 1 card from deck

A - card is Ace

B - card is Spade

P(A) = 4/52 P(B) = 13/52 P(A ∩ B) = 1/52 .

Here P(A ∩ B) = P(A) × P(B) so independent.

In a fully-specified math model, two events are either dependent or independent; can be checked by calculation.

Often we use independence as an assumption in making a model. For instance we assume that different die throws give independent results. Most probability models one encounters in engineering or science have some assumption of “bottom level” independence; but one needs to be careful about which other events within the model are independent.